USING INTERVIEWS TO EXPLORE TEACHER KNOWLEDGE PROFILES IN THE AREA OF PERMUTATIONS

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This paper illustrates consideration of multiple facets of mathematics teacher knowledge through interviews, demonstrates how teachers have different knowledge profiles, and discusses the implications for professional development. This study focuses on interviews with eight teachers, which were analyzed using Ball, Thames, and Phelps' (2008) framework of teacher knowledge, assigning knowledge types to statements made during the interview. All teachers exhibited multifaceted knowledge, but different profiles emerged. The implication is that teacher profiles should be considered in designing professional development. In addition, this analysis supports the use of teacher interviews as a tool to consider professional development needs.

INTRODUCTION

Since the introduction of the idea of pedagogical content knowledge (PCK; Shulman, 1986), studies and theoretical papers have attempted to clarify, specify, or measure Shulman's construct. However, as pointed out by Hill, Ball, and Schilling (2008), there is still little information about how teachers' PCK relates to student-level outcomes, or even about what constitutes PCK. A detailed view of teachers' professional knowledge may help us to enrich it.

This paper considers teacher knowledge to be multifaceted, using the framework of types of teacher knowledge put forth by Ball, Thames, and Phelps (2008), rather than viewing it as unitary. Not all theorized knowledge types have been measured independently or shown to be independent constructs (Hill, Ball, & Schilling, 2008). Thus, they remain, in part, theoretical distinctions. However, even if a different model of teacher knowledge emerges at a later date and is empirically validated, the premise of the analysis in this paper still stands as it points to the importance of recognizing teacher profiles, rather than defining specific profiles a priori.

Here, we are assigning knowledge types to teacher statements made during an interview. In doing so, two themes emerge: (1) teachers' knowledge profiles vary as seen in an interview setting, and (2) these profiles are useful when considering professional development. The study summarizes the results of analyzing eight teacher interviews. In the interviews, the teachers were presented with a mathematical problem and were asked to solve the problem, and also to reflect upon it, explain their solutions, and provide alternative strategies. The mathematical problem asked about how many ways there would be to arrange four distinct objects. The mathematical content, a permutation of all n of n objects [$P(n,n) = n!$] and its influence on responses cannot be

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disregarded; this study does not make claims about what these teachers would do when faced with a different type of problem.

In contrast to written assessments, interviews may provide teachers with more freedom in their responses, afford a more detailed look at their thinking, and serve as a more practical tool for educational administrators or professional development providers seeking to provide targeted development opportunities for teachers.

Defining Teacher Knowledge Types

Shulman (1986) introduced PCK in response to research and standards that heavily emphasized pedagogical procedures divorced from specific content areas. Rejecting this dichotomy, Shulman proposed that teachers needed not content-free pedagogy, nor pedagogy-free content, but a particular kind of professional expertise that went "beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (p. 9). Inside PCK, Shulman included representations, examples, and explanations, as well as common difficulties, common student preconceptions, and ways of changing incorrect student conceptions.

Hill, Ball, and Schilling (2008) give the most comprehensive look at PCK for mathematics. They propose that PCK is part of a larger construct, mathematical knowledge for teaching (MKT) and separate the universe of MKT into (a) subject matter knowledge and (b) pedagogical content knowledge. In this framework, subject matter knowledge includes both common content knowledge (CCK) and specialized content knowledge (SCK). Common content knowledge (or "'common' knowledge of content" in Hill et al., 2005, p. 387) includes what we might consider to be pure mathematical content; this is the knowledge of mathematics apart from the need to teach it. For example, knowing the solution for x in the expression $10^x = 1$. Specialized content knowledge is content knowledge that would be useful only to a teacher; this SCK is still mathematical knowledge, not pedagogy. One example is knowing how to evaluate three methods for multiplying two digit numbers and determine which of the methods are always mathematically valid. This SCK sits next to PCK but does not contain it; neither is it contained by it (Hill et al., 2008). It is knowledge that would be useful while engaged in teaching, but does not require one to know anything about students or about teaching.

PCK includes knowledge of content and students (KCS) and knowledge of content and teaching (KCT). KCS includes "knowledge of how students think about, know, or learn this particular content" (Hill et al., 2008, p. 375). KCT "combines knowing about teaching and knowing about mathematics" (Ball, Thames, & Phelps, 2008, p. 401), that is, knowledge of instructional strategies, choosing examples, and other elements that link the mathematics to the practice of classroom teaching.

Ball et al. (2008) do not limit the types of teacher mathematical knowledge to those described above and leave room in their model for future discovery and definition of knowledge types, particularly as relates to knowledge of the mathematical horizon and knowledge of curriculum. In addition, Ball et al. (2008) and Hill et al. (2008)

acknowledge the difficulty and subtlety in these distinctions, even at a theoretical level. However, the analysis presented here is restricted to these four relatively well-defined knowledge types. To clarify the theory behind these distinctions, Ball et al. (2008) describe examples of tasks in which teachers may engage that would be manifestations of a particular type of knowledge. Their examples paved the way for the analysis carried out in this study.

Accessing Teacher Knowledge Types through Interviews

While teacher knowledge types are theoretical distinctions, they have been described through tasks and measured through written assessment questions (Hill et al., 2005) that attempt to engage respondents in the same types of activities they would be doing as teaching professionals. While teachers may shift fluidly between knowledge types during teaching, and may hold knowledge in complexes (Sherin, 2002), examining the tasks connected to each knowledge type elaborated by Ball et al. (2008) enables us to disentangle the knowledge used.

This paper shifts examination of knowledge types to the interview setting. While Hill et al. (2008) describe conducting interviews as follow up to a written assessment to confirm that teachers were using specific knowledge types, since the interviews in this study will be analyzed statement by statement, the approach, analysis, and results presented here are novel. The interviews elicit teacher knowledge of all types, and the question is how much teachers use each type, and how different profiles emerge.

METHOD

Participants were eight secondary school teachers in a U.S. city or nearby urban rim community participating in a summer professional development workshop who accepted the invitation to be part of this study, which was independent of the workshop. Data were collected through flexible, open-ended individual interviews. The teachers were given problems to solve and, after solving each problem, they were asked for an explanation of their work and then to show a different way to solve the same problem and a different explanation. They were also asked what they believed their students would do when working on the same problem. The analysis presented here focuses on teacher responses to the first problem presented to the teachers. In this problem, teachers were asked how many ways they could arrange four objects. The four objects were presented as characters in boxes. In half of the interviews, the characters were numbers and in half the characters were letters. However, no teacher gave a response specific to either of the two formats, so the answers by all eight teachers were analyzed together.

ANALYSIS

The eight interviews were fully transcribed and statements in each interview were linked to items from the lists of teaching tasks in Ball et al. (2008) for each knowledge type. In each teacher statement, particular tasks were carried out, described, or referred to. A statement was defined as the full length of what a teacher said without response or

interruption from the interviewer. For example, the statement "Yes – after we do a few, most of them see the pattern is that you multiply. So after we do the tree diagram, typically I go into the fundamental counting principle" included four different tasks: Sequence particular content for instruction (KCT); Linking representations to underlying ideas and to other representations (SCK); Anticipate what students are likely to think (KCS); and Anticipate whether students will find a task easy or hard (KCS). Multiple codes were allowed, so more than one knowledge type may have been applied to a single statement, as was the case in this example. The eight teachers produced a total of 168 statements.

RESULTS

In the interviews, all teachers made statements classified under all four knowledge types, exhibiting multifaceted mathematical knowledge for teaching. Considering the eight teachers together, SCK appeared in 55% of statements (see Table 1). The other knowledge types (CCK, KCS, and KCT) appeared in 33% to 35% of statements.

Knowledge type	# of statements linked to this knowledge type (N = 168)	% of statements linked to this knowledge type
CCK	59	35%
SCK	92	55%
KCS	55	33%
KCT	58	35%

Table 1: Relative frequency of statements linked to each knowledge type.

As shown in Table 2, the dominance of SCK varies across individual teachers. SCK, more frequent in the aggregate data, was the most commonly used knowledge type for only four of the eight teachers. In addition, none of the teachers mirrored exactly the profile of the study population as a whole.

Name	**# CCK**	**CCK %**	**# SCK**	**SCK %**	**# KCS**	**KCS %**	**# KCT**	**KCT %**	**HIGHEST %**
Jessica	8 of 14	57%	7 of 14	50%	2 of 14	14%	2 of 14	14%	CCK
Anna	7 of 14	50%	6 of 14	43%	4 of 14	29%	5 of 14	36%	CCK
Sarah	8 of 23	35%	16 of 23	70%	4 of 23	17%	9 of 23	39%	SCK
Whitney	8 of 17	47%	9 of 17	53%	4 of 17	24%	3 of 17	18%	SCK
Annie	6 of 20	30%	16 of 20	80%	7 of 20	35%	3 of 20	15%	SCK
Laura	10 of 35	29%	22 of 35	63%	15 of 35	43%	18 of 35	51%	SCK
Betsy	3 of 18	17%	6 of 18	33%	10 of 18	56%	4 of 18	22%	KCS
Shana	9 of 27	33%	10 of 27	37%	9 of 27	33%	14 of 27	52%	KCT
Total	**59 of 168**	**35%**	**92 of 168**	**55%**	**55 of 168**	**33%**	**58 of 168**	**35%**	**SCK**

Table 2: Percentage of use of knowledge types, by teacher.

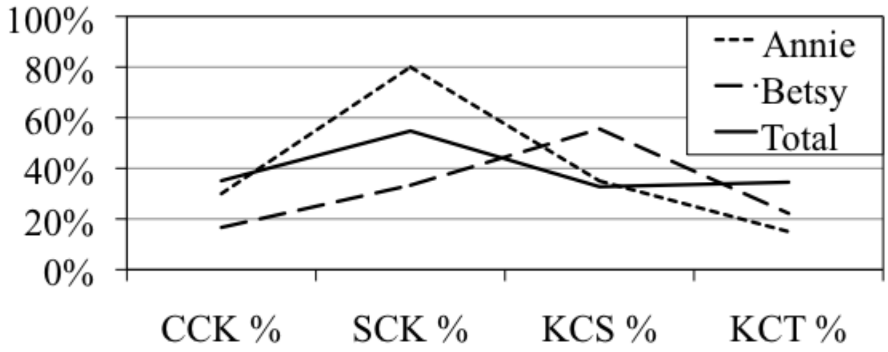

Figure 1: Percentage of statements exhibiting each knowledge type.

Figure 1 provides an image of the percentages for the total and two of the teachers (the data points are connected to easily show all the points corresponding to each teacher). Data on eight teachers are not sufficient to generalize profiles; however, a detailed description may help us begin to construct ideas about the different profiles that could emerge from the teacher knowledge types revealed in interviews. To this goal, two cases will be described here, those of Annie and Betsy. Annie was chosen because she exhibited SCK more than any other knowledge type, mirroring the aggregate data, and because she had the greatest percentage difference between any two knowledge types, with SCK used in 80% of statements and KCT used in only 15%. Betsy was chosen because her profile is at the opposite extreme: she was the only teacher to exhibit KCS more frequently than any other knowledge type, and the only teacher to use KCS more frequently than SCK. These two teacher profiles will enable a more detailed discussion below about the potential for this type of analysis.

Annie, whose work is shown in Figure 2, was a teacher with a strong background in mathematics. She had been teaching for less than five years, but her teaching had always been in secondary school mathematics. When presented with the problem, she quickly found the (correct) answer and explained the procedure she had used. She was able to also talk about two different methods for finding the solution and discuss which one she preferred and why. The use and critique of different representations was a major factor in her high percentage of statements exhibiting SCK, as two of the tasks linked to SCK are "Recognizing what is involved in using a particular representation," and "Linking representations to underlying ideas and to other representations." Annie referred to one (or both) of these tasks in 8 of the 20 statements she made about this interview question, as was the case in this statement:

> We start off with listing them all out, and then do the tree diagram, we can do the tree diagram for it, and then we came up with the formula, so they can see how many choices do they have. And I eventually show them the slots. Like think of 4 chairs that you have and then one person sits here there's only 3 people left, so you take one out.

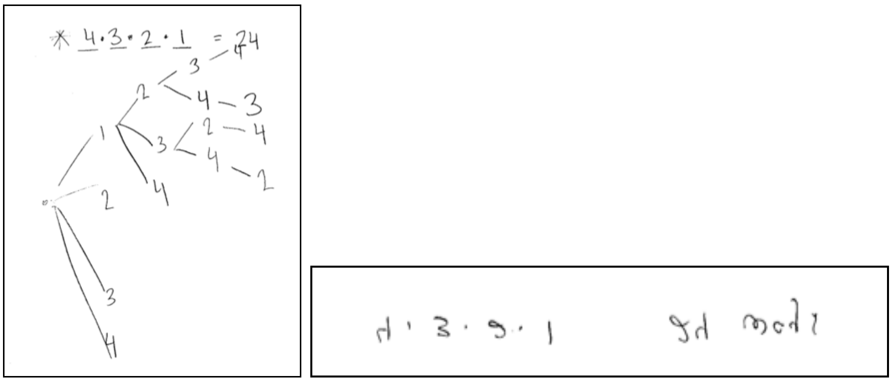

Figure 2: Work on the interview problem. Left: Annie’s; Right: Betsy's.

Other than discussion and use of different representations, the other major factor in Annie’s high percentage of statements exhibiting SCK referred to the task, “Using mathematical notation and language and critiquing its use,” which was identified in 8 of her 20 statements. All of the instances sprung from discussion of how her students would struggle with knowing to multiply, rather than add, the numbers, because of the use of the word “and.” She clarified that students would have trouble “just with the ‘and’ and the ‘or.’ Because doing this stuff [permutations], it means different things.” She elaborated cases when this would occur and how her students would react. Note that these instances of SCK occurred *only* because she was engaged in a task associated with KCS, namely “anticipate what students are likely to think.”

Betsy had been teaching for much longer than Annie had, more than 20 years, but she had not always been a mathematics teacher. She had started by working with special needs students in different subject areas, and then had begun to focus on teaching mathematics with the same population of students at the secondary school level. When Betsy was given the interview problem, she was able to solve it quickly and correctly, as shown in Figure 2, but she was more tentative in her work than Annie was, saying, “Okay, this is the factorial. And granted, I don’t do that too much, but what I understand is you go 4, 3, 2, 1?” When asked about other methods, Betsy was not able to spontaneously think of an alternative, so she did not refer to the same tasks in SCK that Annie had, but she had no difficulty describing how her students would react to the problem and what they would do with similar problems, referring to the tasks “anticipate what students will find confusing” and “anticipate whether students will find a task easy or hard.” For example, when talking about what students would do, Betsy said, “and the other [kind of problem] that they have trouble with too is replacement and without replacement. I mean, some of the kids got it but others just really struggled with it.” Note that by “replacement” and “without replacement,” Betsy meant whether an item could be used again in a permutation once it had already been used once. These terms are common in secondary school classrooms, where they often talk about pulling items from a bag and either replacing or not replacing the selected item before choosing the next.

DISCUSSION

As mentioned above, the analysis of teachers' statements reveals that they all demonstrated all types of knowledge during the part of the interview analyzed in this paper. No teacher had an individual profile that matched the profile of the combined data from all eight participants. What can we make of these wide variations? While making decisions based on these differences now would be unwise, if we were to apply this technique to a larger sample, we might see a set of teacher profiles emerge. In connection with classroom data, we could begin to understand what these different profiles suggest about the teacher's work of teaching.

This is illustrated by the two profiles, Betsy and Annie, described above. We are not naming one profile as superior to the other or preferable for helping students to learn. However, the differences between these two cases illuminate the breadth of experience in mathematics and the variety of perspectives that exist in the teaching force. The analysis of different knowledge types highlights and clarifies the differences between the profiles, and could ultimately help to provide professional support to the teachers. For example, Betsy made relatively few statements showing evidence of SCK. This might lead us to infer that *for this particular mathematical area* (permutations), Betsy could benefit from working in professional development activities related to SCK, such as working with and connecting a variety of representations. Conversely, Annie made few statements that showed evidence of KCT. She might be better supported, then, by professional development that focused on the teaching aspect, such as choosing examples or deciding how to respond to student contributions. Another advantage of examining these teacher profiles is that we begin to see that different profiles may complement each other. That is, perhaps Betsy and Annie would be able to each take the lead in turn in sharing teaching knowledge with each other in a mutually beneficial way.

This particular interview analysis is different from previous work on distinguishing teacher knowledge types. The analysis of individual statements in interviews is based upon Ball et al. (2008), but not recommended or endorsed by them. While not as easy to code, the interview allows for a more descriptive view of a teacher's varied knowledge. This may help us not only to understand the different teacher profiles, but also to begin to see how they complement each other, as in the cases described above. In addition, written assessments that can claim to measure a particular type of teacher knowledge, like that described by Hill et al. (2005), need to be developed and tested extensively. By necessity, they can only cover a finite number of mathematical topics. If we want to know more about teacher knowledge about something specific, like the permutation question analyzed above, a coded interview allows this targeted examination. Interested researchers and those who work on professional development could look at teacher knowledge in their particular mathematical domain, even when they do not possess the resources that would be required to develop a written assessment.

It is important to consider that the freedom of an interview may make it more likely that teachers will elaborate on the elements that interest them. In doing so, they move back and forth quite fluidly between knowledge types. This is supported by the findings above that all eight teachers exhibited all four knowledge types. In fact, Sherin (2002) suggests that teachers may access “content knowledge complexes” (p. 124), where the teachers’ past experience creates a link between the content and the pedagogy that results in accessing these types of knowledge together. The way that the teachers in this study moved easily between knowledge types lends support to Sherin’s theory. However, using the terms put forth by Shulman (1986), she says, “I claim that there are larger elements of teacher knowledge that cannot be categorized either as subject matter knowledge or as pedagogical content knowledge” (Sherin, 2002, p. 124-125.) We would suggest instead that it is not that a complex exhibited by a teacher can be classified as *neither* type of knowledge, but rather that it can be classified as *more than one* type of knowledge. The idea of content knowledge complexes gives us a view of how different knowledge is called forth by a teacher, but it does not preclude us from categorizing teacher statements more specifically.

While mapping and coding knowledge types may begin as a theoretical exercise, it is one with a practical goal. A unitary approach to professional development for teachers ignores their varying knowledge profiles. While it may seem obvious that teachers differ, we know little about how to determine which tasks they need support with and how to provide this support. Careful examination of their profiles through interviews about the complex tasks of teaching can only help.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Hill, H. C., Ball, D. L, & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers’ topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372-400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction*, *20*(2), 119-150.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.